by the wire. The satellite was watched several minutes before and after the conjunction. On Dec. 3 *Mimas* was observed in conjunction with the preceding end of the Cassini division of the ring, and again when in conjunction with the preceding edge of the ball. On good nights, with the present opening of the ring, this satellite can be followed to conjunction with ease and certainty.

The two observations of the inner satellite of *Mars* were made with difficulty, and these observations are of little value except to show that the elements of this satellite are nearly correct. The outer satellite could be observed accurately for about one month when near opposition. It could be followed longer with our 26-inch glass, but it is hardly worth while to do this, and thus bring in a quantity of uncertain observations. In both cases the residuals in the angles indicate small corrections to the periods of these satellites, but these periods are no doubt accurate enough to carry forward the Ephemerides for several succeeding oppositions of the planet. As the motion of these satellites in angle of position is now direct, or different from what it was at the time of discovery, it seems better to wait for further observations—at least during one favourable opposition—before making a final discussion of the observations. The observations above are corrected for differential refraction, and for the figure of the planet.

I am much indebted to Mr. Marth for the Ephemerides of satellites that he publishes.

The companion of *Sirius* has been very faint and difficult to observe during the present year. This was owing chiefly, I think, to the very unfavourable weather that has prevailed. The rainfall at Washington has been about 21 inches during the first three months of the year.

The Motion of Hyperion. By Prof. Asaph Hall.

In the *Astronomische Nachrichten*, Nos. 2246 and 2263, I have called attention to the motion of this satellite of *Saturn*, and to the difficulties that prevent an accurate determination of this motion. These difficulties arise in part from the fewness of the observations made from the time of its discovery in 1848 until 1875. In fact, during this interval of twenty-seven years the only complete series of observations is that made by Mr. Lassell at the opposition of 1852. For this reason the mean motion of this satellite is not yet well known, and it appears that we must wait for still further observations; but the following results may be of some interest, and may induce some astronomer who has the means, to assist in observing this faint object.

Denoting by s the observed distance of the satellite from the

centre of *Saturn*, and by p the corresponding angle of position, the formulæ of Marth give

$$r \sin (\mathrm{H}+u) = \frac{\rho}{\rho_0} s . \frac{\cos (p-k)}{\cos h}$$

$$r \cos (\mathrm{H}+u) = \frac{\rho}{\rho_0} s . \sin (p-k)$$

In these formulæ r is the radius vector of the satellite, and u is the argument of the latitude, ρ is the distance of the planet from the Earth, and ρ_0 its mean distance from the Sun. The quantities h, k, and $\cos h$ depend on the position of the orbit plane of the satellite, and the Right Ascension and Declination of the planet; and hence these quantities may be tabulated when the inclination and node of the orbit plane with respect to the equator are given. The plane of the orbit of *Hyperion* coincides nearly with that of *Titan*, and its position might be assumed the same, but I have used my former determination of the node and inclination which does not differ much from those of *Titan*. The quantities h, k, and $\cos h$ were then tabulated for each of the oppositions 1875 to 1883, and the values of r and u were computed from the observed values of s and p. Then having the radii vectores and the angles u, which vary nearly as the true anomalies, the elements of the satellite in the plane of its orbit were found. The following table gives the results of this calculation. The first column gives the year of the opposition, and I have included Mr. Lassell's observations of 1852. The second column gives the number of equations of condition. Each observation furnishes an equation of the form

$$x + by + cz + n = 0,$$

where

$$b = r \cos u,\ c = r \sin u,\ n = p_0 - r,$$
$$x = \Delta p_0,\ y = -e \cos (\mathrm{N}-\mathrm{P}),\ z = e \sin (\mathrm{N}-\mathrm{P});$$

and p_0 is an assumed value of the semi-parameter, N and P are the longitudes of the node and peri-saturnium on the equator, and e is the eccentricity of the orbit. These equations solved by the method of least squares gave the quantities in the following columns. The third and fourth columns give the resulting values of the semi-major axis, and the eccentricity of the orbit. The fifth column gives the value of the longitude of the peri-saturnium with respect to the equator. In the last column is given the average value of $\cos h$. It will be seen by the formulæ that $\cos h$ enters into the value of $r \sin (\mathrm{H}+u)$, and when this cosine is very small the values of r and u may be largely affected by small errors of observation. For this reason I have omitted the years 1877 and 1878, when the value of $\cos h$ was less than one-tenth.

Date.	No. of Equations.	a	e	P	cos h.
1852·9	30	$217''.05$	0·1201	$240^\circ.182$	0·33
1875·7	32	216·25	0·1026	174·242	0·23
1876·7	11	216·52	0·1290	156·922	0·15
1879·8	27	211·17	0·0780	93·173	0·13
1880·9	48	212·41	0·0823	60·012	0·23
1881·9	42	213·05	0·0898	36·908	0·30
1882·9	15	215·46	0·0884	20·043	0·37
1883·9	30	212·90	0·0982	353·272	0·41

This table shows a very large change in the value of P. These values may be represented very nearly by the formula

$$P = 174^\circ.242 - 20^\circ.344t - 0^\circ.10254t^2$$

t denoting the number of Julian years from the epoch 1875–7.

There is yet some uncertainty in the mean motion of *Hyperion*, but the best value is found probably by comparing Mr. Lassell's observations of 1852 with the recent ones. A comparison of these observations with my own of 1880 and 1882 gives the following value of the periodic time:

$$\text{Sidereal Period} = 21{\cdot}276742 \text{ days};$$

and the synodical period is 21·318901 days.

On account of its mean distance and the eccentricity of its orbit *Hyperion* can approach very near to *Titan*; and since three times the period of one is nearly equal to four times the period of the other, there is probably a large perturbation of the motion of *Hyperion* by *Titan*, depending on the first powers of the eccentricities of their orbits. With the above value of the mean motion of *Hyperion* this perturbation will have a period of 19·185 years. The complete calculation of such a perturbation would be laborious in this case, since in computing the $b_s^{(i)}$ coefficients of Laplace we shall have

$$\alpha = \frac{176{\cdot}552}{213{\cdot}98} = 0{\cdot}8250863,$$

and the series for these coefficients converge slowly.

Also a small change in the mean motion of *Hyperion* would have a large influence on the values of the coefficients in the expressions for the perturbations, and for this reason it does not seem worth while to give much time to such a work at present. But, in order to form some idea of the action of *Titan*, I have computed the coefficients in the perturbations of the radius vector and the true longitude, including the first powers of the eccentricities by the formulæ of the *Mécanique Céleste*, tome i. pp. 279, 280. In computing the values of $b_{\frac{1}{2}}^{(i)}$ and the derivatives, no attempt was made to carry the numerical approximations very far. Distinguishing the elements of *Titan* with an accent we have

$$
\begin{array}{ll}
a = 213''{\cdot}98 & a' = 176''{\cdot}552 \\
e = \quad 0{\cdot}1 & e' = \quad 0{\cdot}029223 \\
\tau = \quad 21{\cdot}276742 \text{ days} & \tau' = \quad 15{\cdot}94545154 \text{ days} \\
 & m' = \text{ mass of Titan.}
\end{array}
$$

$$
\delta r = am' . \left\{
\begin{array}{l}
-\ 0{\cdot}32865 -\ \ 4{\cdot}664 . \cos\,(n't - nt + \epsilon' - \epsilon) \\
\qquad\qquad\quad -\ \ 2{\cdot}902 . \cos 2\,(n't - nt + \epsilon' - \epsilon) \\
\qquad\qquad\quad +183{\cdot}075 . \cos 3\,(n't - nt + \epsilon' - \epsilon) \\
\qquad\qquad\quad +\ \ 0{\cdot}114 . \cos 4\,(n't - nt + \epsilon' - \epsilon) \\
+\ \ 0{\cdot}407 . \cos\,(nt + \epsilon - \pi) - 0{\cdot}1607 . \cos\,(nt + \epsilon - \pi') \\
+\ \ 6{\cdot}332 . t \sin\,(nt + \epsilon - \pi) - 2{\cdot}0570 . t \sin\,(nt + \epsilon - \pi') \\
-166{\cdot}367 . \cos\,[(4n - 3n') . t + 4\epsilon - 3\epsilon - \pi] \\
+\ 16{\cdot}814 . \cos\,[(4n - 3n') . t + 4\epsilon - 3\epsilon' - \pi']
\end{array}
\right\}
$$

$$
\delta v = m' . \left\{
\begin{array}{l}
\qquad\qquad\quad +\ 30{\cdot}065 . \sin\,(n't - nt + \epsilon' - \epsilon) \\
\qquad\qquad\quad +\ \ 5{\cdot}739 . \sin 2\,(n't - nt + \epsilon' - \epsilon) \\
\qquad\qquad\quad -366{\cdot}667 . \sin 3\,(n't - nt + \epsilon' - \epsilon) \\
\qquad\qquad\quad -\ \ 0{\cdot}932 . \sin 4\,(n't - nt + \epsilon' - \epsilon) \\
-12{\cdot}665 . t \cos\,(nt + \epsilon - \pi) + 4{\cdot}114 . t \cos\,(nt + \epsilon - \pi') \\
-73156{\cdot}67 \sin\,[(4n - 3n')t + 4\epsilon - 3\epsilon' - \pi] \\
+8307{\cdot}50 \sin\,[(4n - 3n')t + 4\epsilon - 3\epsilon' - \pi']
\end{array}
\right\}
$$

The large coefficients in the perturbation of the longitude indicate that a small mass of *Titan* may have a great influence on the motion of *Hyperion*. But the mean motion of this last satellite must be known more exactly before any inference can be drawn. It is possible that the quantity $4n - 3n'$ may prove to be extremely small, so that we may have here a relation similar to that which exists among the satellites of *Jupiter*. Should such be the case the above values of δr and δv would be of little use, and the theory would be greatly changed. The mass of *Titan* is known only from estimates of its diameter, and by assuming its density. On several occasions during the past eight years, when the images were good, I have estimated the apparent diameter of *Titan* to be $0''{\cdot}75$; but looking at some of the fixed stars on the same nights these also appeared to have small disks, and hence I am distrustful of diameters estimated in this way. According to Professor Pickering's photometric measurements, and assuming the densities of these satellites to be the same, we have

Mass of Titan = 387 times mass of Hyperion.

It is possible now, I think, to make a tolerably accurate ephemeris of *Hyperion* for the next opposition, but the one given in the American *Ephemeris* will serve very well for observing the satellite. It should be stated, however, that during the

opposition of 1883, *Hyperion* came to conjunction about one day sooner than the times given in that *Ephemeris.*

The observations of *Hyperion* made by Mr. Lassell during the opposition of 1852 are important, since they form the earliest complete series that we have. These observations are published in the *Monthly Notices*, vol. xiii. p. 181. I hope that Mr. Marth or some one familiar with Mr. Lassell's method of work, may be willing to revise the reductions and publish the observations with more detail.

1884 *April* 10.

Observations of the Companion of Sirius, made at the Dearborn Observatory, Chicago, U.S.A. By Prof. G. W. Hough and S. W. Burnham.

Date.	p.	s.
1884·025	37°·9	8″·60
·057	36·7	8·35
·170	37·0	8·64
·186	37·1	8·39
·197	36·5	8·78
·200	36·5	8·58
·203	36·9	8·37
·217	37·0	8·51
·225	36·1	8·51
·233	35·9	8·30
·252	35·9	8·58
Mean = 1884·179	36·7	8·51

(*Observations by S. W. Burnham.*)

Date.	p.	s.
1884·057	37°·2	8″·60
·167	39·3	8·33
·184	33·2	8·56
·195	36·4	8·11
·197	35·3	8·50
·200	35·8	8·31
·214	37·0	8·34
·217	36·3	8·22
·230	37·9	8·44
·233	35·5	8·51
Mean = 1884·19	36·4	8·39

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